Lorentz-CPT violation, radiative corrections and finite temperature

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Abstract: In this work we investigate the radiatively induced Chern-Simons-like terms in four-dimensions at zero and finite temperature. We use the approach of rationalizing the fermion propagator up to the leading order in the CPT-violating coupling b_μ. In this approach, we have shown that although the coefficient of Chern-Simons term can be found unambiguously in different regularization schemes at zero or finite temperature, it remains undetermined. We observe a correspondence among results obtained at finite and zero temperature.

Keywords: Chern-Simons Theories, Space-Time Symmetries, Thermal Field Theory.

Contents

1. Introduction

In recent years, it has been investigated in the literature the possibility of the Lorentz and CPT symmetries being violated in the nature [1 – 16]. Theoretical investigations have pointed out that these symmetries can be approximate. The realization of this violation can be obtained in QED by adding the Chern-Simons-like term with a constant quadrivector k_μ, $\frac{1}{2}k_\mu \epsilon^{\mu\alpha\beta\gamma}F_{\alpha\beta}A_\gamma$, to the Maxwell's theory, and another term which is a CPT-odd term for fermions, i.e., $\bar{\psi}\slashed{b}\gamma_5\psi$ with a constant quadrivector b_μ. This extension of the QED does not break the gauge symmetry of the action and equations of motion but it does modify the dispersion relations for different polarization of photons and Dirac's spinors. Interesting investigations in the context of Lorentz-CPT violation have appeared recently in the literature, where several issues were addressed, such as Čerenkov-type mechanism called "vacuum Čerenkov radiation" to test the Lorentz symmetry [17], changing of gravitational redshifts for differently polarized Maxwell-Chern-Simons photons [18], evidence for the Lorentz-CPT violation from the measurement of CMB polarization [19], supersymmetric extensions [20], breaking of the Lorentz group down to the little group associated with k_μ [21] and magnetic monopoles inducing electric current [22].

The dynamical origin of the parameters k_μ and b_μ present in the Lorentz and CPT symmetry breaking is obtained when we integrate over the fermion fields in the modified Dirac action such that the radiative corrections may lead to $k_\mu = Cb_\mu$. Several studies have shown that C can be found to be finite but undetermined [23 – 27].

In the present work, we focus attention on induced Chern-Simons-like terms via radiative corrections both in zero and finite temperature by using different regularization schemes. Our starting point here is the use of the approach of rationalizing the fermion propagator up to the leading order in the CPT-violating coupling b_μ, after using the derivative expansion method. By using this approach we have obtained new and previous results of the literature and also observe the existence of new effects at finite temperature. Then, by using dimensional regularization and Lorentz invariant regularization schemes at zero temperature, we show that the Chern-Simons coefficient is found to be unambiguously

finite, but with different values. We found that these coefficients are indeed limits of the theory when we take into account finite temperature and only use the Lorentz invariant regularization and derivative expansion method. On the other hand, by using dimensional regularization at finite temperature, in our present approach, leads to other finite results in zero and finite temperature found in the literature.

2. Radiative corrections

The one-loop effective action $S_{\text{eff}}[b, A(x)]$ of the gauge field $A(x)$ can be expressed in the form of the following functional trace

$$S_{\text{eff}}[b, A(x)] = -i \operatorname{Tr} \ln(\not{p} - m - \not{b}\gamma_5 - e\not{A}(x)). \tag{2.1}$$

This functional trace can be represented as $S_{\text{eff}}[b, A(x)] = S_{\text{eff}}[b] + S'_{\text{eff}}[b, A(x)]$. The first term $S_{\text{eff}}[b] = -i \operatorname{Tr} \ln(\not{p} - m - \not{b}\gamma_5)$ does not depend on the gauge field, and the only nontrivial dynamics is concentrated in the second term $S'_{\text{eff}}[b, A(x)]$, which is given by the following power series

$$S'_{\text{eff}}[b, A(x)] = i \operatorname{Tr} \sum_{n=1}^{\infty} \frac{1}{n} \left[\frac{1}{\not{p} - m - \not{b}\gamma_5} \, e\not{A}(x) \right]^n. \tag{2.2}$$

To obtain the Chern-Simons term we should expand this expression up to the second order in the gauge field

$$S'_{\text{eff}}[b, A(x)] = S^{(2)}_{\text{eff}}[b, A(x)] + \cdots, \tag{2.3}$$

where

$$S^{(2)}_{\text{eff}}[b, A(x)] = \frac{-ie^2}{2} \operatorname{Tr} \left[S_b(p) \, \not{A}(x) \, S_b(p) \, \not{A}(x) \, \right], \tag{2.4}$$

with $S_b(p)$ being the exact fermion propagator expressed in the form

$$S_b(p) = \frac{i}{\not{p} - m - \not{b}\gamma_5}. \tag{2.5}$$

Using the derivative expansion method [28] one can find that the first one-loop contribution to $S^{(2)}_{\text{eff}}[b, A]$ reads

$$S^{(2)}_{\text{eff}}[b, A(x)] = \frac{1}{2} \int d^4x \, \Pi^{\alpha\mu\nu} \partial_\alpha A_\mu A_\nu, \tag{2.6}$$

where the one-loop self-energy $\Pi^{\alpha\mu\nu}$ is given by

$$\Pi^{\alpha\mu\nu} = -ie^2 \int \frac{d^4p}{(2\pi)^4} \operatorname{tr} \left[S_b(p) \, \gamma^\mu \, S_b(p) \, \gamma^\alpha \, S_b(p) \, \gamma^\nu \right]. \tag{2.7}$$

We now focus on the study of possible indetermination of the Chern-Simons coefficient that can appear when we expand the self-energy (2.7) as in the approach of rationalizing the fermion propagator up to the leading order in b. Then, we may use the approximation scheme developed in [29] to expand the exact propagator in eq. (2.5) up to the first order in b

$$S_b(p) = i \left[\frac{\not{p} + m - \gamma_5 \not{b}}{(p^2 - m^2)} - \frac{2\gamma_5 (m\not{b} - (b \cdot p))(\not{p} + m)}{(p^2 - m^2)^2} \right] + \cdots, \tag{2.8}$$

where we have retained only the leading order terms in b. Before everything, let us use the following notation

$$S_b(p) = i(W_1 + W_2\gamma_5), \tag{2.9}$$

where

$$W_1 = \frac{\not{p} + m}{(p^2 - m^2)}, \tag{2.10}$$

$$W_2 = \frac{\not{b}}{(p^2 - m^2)} - \frac{\left[2(m\not{b} + (b \cdot p))(\not{p} - m)\right]}{(p^2 - m^2)^2}, \tag{2.11}$$

into eq. (2.7) to get the form

$$\Pi^{\mu\alpha\nu} = -e^2 \int \frac{d^4p}{(2\pi)^4} \mathrm{tr}\left[W_1\gamma^\mu W_1\gamma^\alpha W_2\gamma_5\gamma^\nu + W_1\gamma^\mu W_2\gamma_5\gamma^\alpha W_1\gamma^\nu \right. $$
$$\left. + W_2\gamma_5\gamma^\mu W_1\gamma^\alpha W_1\gamma^\nu\right]. \tag{2.12}$$

Substituting (2.10) and (2.11) into (2.12) and using the relation $\{\gamma^\mu, \gamma_5\} = 0$, we can calculate the trace of gamma matrices, resulting in the following expression

$$\Pi^{\mu\alpha\nu} = 4ie^2 \int \frac{d^4p}{(2\pi)^4} \frac{1}{(p^2 - m^2)^3} \{3\varepsilon^{\alpha\mu\nu\theta}\left[b_\theta(p^2 + m^2) - 2p_\theta(b \cdot p)\right] + $$
$$- 2b_\theta\left[\varepsilon^{\beta\mu\nu\theta}p_\beta p^\alpha + \varepsilon^{\alpha\beta\nu\theta}p_\beta p^\mu + \varepsilon^{\alpha\mu\beta\theta}p_\beta p^\nu\right]\}. \tag{2.13}$$

Note that by power counting, the momentum integral in (2.13) involves finite terms and terms with logarithmic divergence. Because the integral is divergent, there is no unique answer regardless of the regularization scheme used, e.g., Pauli-Villars or dimensional regularization [23]. Here, we shall adopt simpler regularization schemes in order to calculate the divergent integral, such as dimensional regularization [30] and Lorentz preserving regularization. The above integral is promoted to D dimensions and a straightforward calculation yields

$$\Pi^{\mu\alpha\nu} = \frac{6e^2}{(4\pi)^{D/2}} \frac{\epsilon\,\Gamma(\epsilon/2)}{(m^2)^{\epsilon/2}} \varepsilon^{\mu\alpha\nu\theta} b_\theta, \tag{2.14}$$

where $\epsilon = 4 - D$. Therefore, for $D = 4$, we find

$$S_{\mathrm{eff}}^{(2.2)}[b, A(x)] = \frac{3e^2}{8\pi^2} \int d^4x\, b_\beta \varepsilon^{\mu\alpha\nu\beta}\partial_\alpha A_\mu A_\nu. \tag{2.15}$$

This shows that the Chern-Simons coefficient k_β relates with b_β is the form

$$k_\beta = \frac{3e^2}{8\pi^2} b_\beta. \tag{2.16}$$

On the other hand, there also exists the possibility of using in eq. (2.13) the relation

$$\int \frac{d^4p}{(2\pi)^4} p_\mu p_\nu\, f(p^2) = \frac{g_{\mu\nu}}{4} \int \frac{d^4p}{(2\pi)^4} p^2\, f(p^2), \tag{2.17}$$

that naturally removes the logarithmic divergence. As a result, we have only the finite contribution

$$\Pi^{\mu\alpha\nu} = \frac{6e^2}{(4\pi)^{D/2}} \frac{\Gamma(1+\epsilon/2)}{(m^2)^{\epsilon/2}} \varepsilon^{\mu\alpha\nu\theta} b_\theta. \tag{2.18}$$

Therefore, for $D = 4$, we find

$$S_{\text{eff}}^{(2.2)}[b, A(x)] = \frac{3e^2}{16\pi^2} \int d^4x \, b_\beta \varepsilon^{\mu\alpha\nu\beta} \partial_\alpha A_\mu A_\nu, \tag{2.19}$$

which shows that the Chern-Simons coefficient k_β now relates with b_β in the form

$$k_\beta = \frac{3e^2}{16\pi^2} b_\beta. \tag{2.20}$$

Although the results (2.16) and (2.20) were obtained unambiguously in different regularization schemes, they show that k_β is indeed undetermined. One the other hand, it is interesting to note that these results were shown to be connected to each other at finite temperature [31]. The results (2.16) and (2.20) are limits of high temperature and zero temperature, respectively, as it was obtained in [31] where the logarithmic divergences were eliminated by using the Lorentz preserving regularization (2.17).

3. Finite temperature effects

Let us now study the Chern-Simons coefficient when we take temperature into account. The effect of high temperature in the context of breaking Lorentz and CPT symmetries has generated interesting studies in the literature [29, 31–34]. However, only [29] uses the approach (2.8) of rationalizing the fermion propagator up to the leading order in b in the context of finite temperature. Following such an approach, in this section, we are going to investigate the Chern-Simons coefficient at finite temperature.

To develop calculations with finite temperature, let us now assume that the system is in the state of the thermal equilibrium with a temperature $T = 1/\beta$. In this case we can use the Matsubara formalism for fermions. This consists of taking $p_0 \equiv \omega_n = (n + 1/2)\frac{2\pi}{\beta}$ and replacing the integration over zeroth component of the momentum by a discrete sum $(1/2\pi) \int dp_0 \to \frac{1}{\beta} \sum_n$ [35]. We also change the Minkowski space to Euclidean space by performing the Wick rotation $x_0 \to -ix_0$, $p_0 \to ip_0$, $b_0 \to ib_0$, $d^4x \to -id^4x$ and $d^4p \to id^4p$. Considering initially the expression (2.13), we have that the effect of the temperature must reproduce the following structure

$$\Pi^{\mu\alpha\nu} = \frac{-4ie^2}{\beta} \sum_{n=-\infty}^{\infty} \int \frac{d^3\vec{p}}{(2\pi)^3} \frac{1}{(\vec{p}^2 + M_n^2)^3} \{3\varepsilon^{\alpha\mu\nu\theta} \left[b_\theta(\vec{p}^2 + M_n^2 - 2m^2) - 2p_\theta(b \cdot p)\right] + $$
$$-2b_\theta \left[\varepsilon^{\beta\mu\nu\theta} p_\beta p^\alpha + \varepsilon^{\alpha\beta\nu\theta} p_\beta p^\mu + \varepsilon^{\alpha\mu\beta\theta} p_\beta p^\nu\right]\}, \tag{3.1}$$

where

$$M_n^2 = \left(n + \frac{1}{2}\right)^2 \frac{4\pi^2}{\beta^2} + m^2. \tag{3.2}$$

To implement translation only on the space coordinates of the loop momentum p_ρ we decompose it as follows [33]

$$p_\rho \rightarrow \vec{p}_\rho + p_0 \delta_{0\rho}. \tag{3.3}$$

We use the covariance under spatial rotations which allows us to carry out the following replacement

$$\vec{p}_\rho \vec{p}^\sigma \rightarrow \frac{\vec{p}^2}{D}(\delta_\rho^\sigma - \delta_{\rho 0}\delta_0^\sigma). \tag{3.4}$$

Thus,

$$
\begin{aligned}
2p_\rho b_\sigma p^\sigma &\rightarrow 2\left(b_\rho \frac{\vec{p}^2}{D} - b_0 \delta_{\rho 0}\left(\frac{\vec{p}^2}{D} - p_0^2\right)\right), \\
2p_\beta p^\alpha &\rightarrow 2\left(\delta_\beta^\alpha \frac{\vec{p}^2}{D} - \delta_{\beta 0}\delta_0^\alpha\left(\frac{\vec{p}^2}{D} - p_0^2\right)\right), \\
2p_\beta p^\mu &\rightarrow 2\left(\delta_\beta^\mu \frac{\vec{p}^2}{D} - \delta_{\beta 0}\delta_0^\mu\left(\frac{\vec{p}^2}{D} - p_0^2\right)\right), \\
2p_\beta p^\nu &\rightarrow 2\left(\delta_\beta^\nu \frac{\vec{p}^2}{D} - \delta_{\beta 0}\delta_0^\nu\left(\frac{\vec{p}^2}{D} - p_0^2\right)\right).
\end{aligned} \tag{3.5}
$$

We have that only the terms above can contribute to the Chern-Simons structure. Therefore, we can split the expression (2.6) into a sum of two parts, "covariant" and "noncovariant", i.e.,

$$S_{\text{eff}}^{\text{cov}} = (-i)\int d^4x\, I_1(m,\beta)\varepsilon^{\alpha\mu\nu\beta}\, b_\beta \partial_\alpha A_\mu A_\nu, \tag{3.6}$$

with

$$I_1(m,\beta) = \frac{-6ie^2}{\beta}\sum_{n=-\infty}^{\infty}\int \frac{d^D\vec{p}}{(2\pi)^D}\frac{(1-\frac{4}{D})\vec{p}^2 + M_n^2 - 2m^2}{(\vec{p}^2 + M_n^2)^3}, \tag{3.7}$$

and

$$S_{\text{eff}}^{\text{ncv}} = i\int d^4x\, I_2(m,\beta)\left[3\varepsilon^{\alpha\mu\nu 0}\, b_0 \partial_\alpha A_\mu A_\nu + \right.$$
$$\left. + b_\theta(\varepsilon^{0\mu\nu\theta}\partial_0 A_\mu A_\nu + \varepsilon^{\alpha 0\nu\theta}\partial_\alpha A_0 A_\nu + \varepsilon^{\alpha\mu 0\theta}\partial_\alpha A_\mu A_0)\right], \tag{3.8}$$

with

$$I_2(m,\beta) = \frac{4ie^2}{\beta}\sum_{n=-\infty}^{\infty}\int \frac{d^D\vec{p}}{(2\pi)^D}\frac{\frac{\vec{p}^2}{D} - M_n^2 + m^2}{(\vec{p}^2 + M_n^2)^3}. \tag{3.9}$$

After integration over the spatial momentum, we find

$$
\begin{aligned}
I_1(m,\beta) &= \frac{-3ie^2}{(4\pi)^{D/2}\beta}\sum_{n=-\infty}^{\infty}\left[\frac{(\Gamma(3-\frac{D}{2}) - (2-\frac{D}{2})\Gamma(2-\frac{D}{2}))}{(M_n^2)^{2-\frac{D}{2}}} - \frac{2m^2\Gamma(3-\frac{D}{2})}{(M_n^2)^{3-\frac{D}{2}}}\right], \\
&= \frac{6im^2e^2\Gamma(\lambda_2)}{(4\pi)^{D/2}\beta}\left(\frac{a^2}{m^2}\right)^{\lambda_2}\sum_{n=-\infty}^{\infty}\frac{1}{((n+b)^2 + a^2)^{\lambda_2}},
\end{aligned} \tag{3.10}
$$

and

$$I_2(m, \beta) = \frac{ie^2}{(4\pi)^{D/2}\beta} \sum_{n=-\infty}^{\infty} \left[\frac{\epsilon'\Gamma(2-\frac{D}{2})}{(M_n^2)^{2-\frac{D}{2}}} - \frac{2m^2\Gamma(3-\frac{D}{2})}{(M_n^2)^{3-\frac{D}{2}}} \right],$$

$$= \frac{ie^2}{(4\pi)^{D/2}\beta} \sum_{n=-\infty}^{\infty} \left[\left(\frac{a^2}{m^2}\right)^{\lambda_1} \frac{\epsilon'\Gamma(\lambda_1)}{((n+b)^2+a^2)^{\lambda_1}} - \left(\frac{a^2}{m^2}\right)^{\lambda_2} \frac{2m^2\Gamma(\lambda_2)}{((n+b)^2+a^2)^{\lambda_2}} \right] \quad (3.11)$$

where $\lambda_1 = 2 - \frac{D}{2}$, $\lambda_2 = 3 - \frac{D}{2}$, $\epsilon' = 3 - D$, $a = m\beta/2\pi$ and $b = 1/2$.

At this point we need an explicit expression for the sum over the Matsubara frequencies. We use the following result [36]

$$\sum_n \frac{1}{[(n+b)^2+a^2]^\lambda} = \frac{\sqrt{\pi}\Gamma(\lambda-1/2)}{\Gamma(\lambda)(a^2)^{\lambda-1/2}} +$$
$$+ 4\sin(\pi\lambda) \int_{|a|}^{\infty} \frac{dz}{(z^2-a^2)^\lambda} Re\left(\frac{1}{\exp 2\pi(z+ib)-1}\right), \quad (3.12)$$

which is valid for $1/2 < \lambda < 1$. Note that for $\lambda_1 = 2 - \frac{D}{2}$ and $\lambda_2 = 3 - \frac{D}{2}$ we cannot apply this relation for $D = 3$ since the integral diverges. Thus, we carry out the analytic continuation of this relation, so that we obtain

$$\int_{|a|}^{\infty} \frac{dz}{(z^2-a^2)^\lambda} Re\left(\frac{1}{\exp 2\pi(z+ib)-1}\right) =$$
$$= \frac{1}{2a^2} \frac{3-2\lambda}{1-\lambda} \int_{|a|}^{\infty} \frac{dz}{(z^2-a^2)^{\lambda-1}} Re\left(\frac{1}{\exp 2\pi(z+ib)-1}\right) \quad (3.13)$$
$$- \frac{1}{4a^2} \frac{1}{(2-\lambda)(1-\lambda)} \int_{|a|}^{\infty} \frac{dz}{(z^2-a^2)^{\lambda-2}} \frac{d^2}{dz^2} Re\left(\frac{1}{\exp 2\pi(z+ib)-1}\right).$$

Now we can substitute this expression into (3.12), for $D = 3$, so that after some simplifications we get

$$I_1(\beta) = \frac{3ie^2}{8\pi^2}[1 + 2\pi^2 F(a)], \quad (3.14)$$

and

$$I_2(\beta) = -\frac{ie^2}{4} F(a), \quad (3.15)$$

where the function $F(a)$ given by

$$F(a) = \int_{|a|}^{\infty} dz(z^2-a^2)^{1/2} \frac{\tanh(\pi z)}{\cosh^2(\pi z)}, \quad (3.16)$$

has the following limits: $F(a \to \infty) \to 0$ $(T \to 0)$ and $F(a \to 0) \to 1/2\pi^2$ $(T \to \infty)$ — see figure 1.

In summary we find that the Chern-Simons coefficients at finite temperature for both covariant and noncovariant parts are:
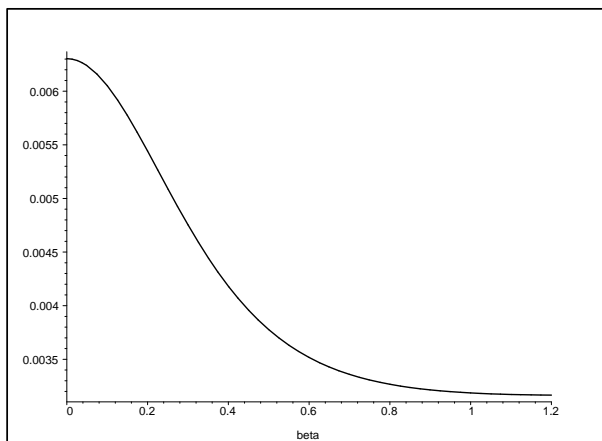
Figure 1: The function $F(a)$ is diferent from zero everywhere. At zero temperature ($\beta \to \infty$), the function tends to a nonzero value.

$$k_\alpha^{\mathrm{cov}} = \frac{3e^2}{4\pi^2} b_\alpha, \qquad k_0^{\mathrm{ncv}} = \frac{3e^2}{8\pi^2} b_0, \qquad k_i^{\mathrm{ncv}} = \frac{e^2}{8\pi^2} b_i, \quad (a \to 0 \text{ or } T \to \infty) \quad (3.17)$$

$$k_\alpha^{\mathrm{cov}} = \frac{3e^2}{8\pi^2} b_\alpha, \qquad k_\alpha^{\mathrm{ncv}} = 0. \hspace{4cm} (a \to \infty \text{ or } T \to 0) \quad (3.18)$$

Note that the covariant coefficient $\frac{3e^2}{8\pi^2} b_\alpha$ at $T \to 0$ coincides with the coefficient previously obtained at zero temperature (2.16), and corresponds to the result obtained in [31] at the limit $T \to \infty$. The noncovariant result $\frac{e^2}{8\pi^2} b_i$ at $T \to \infty$ corresponds to the result found in [13, 37] at zero temperature, and the noncovariant result $k_\alpha^{\mathrm{ncv}} = 0$ at $T \to 0$ corresponds to the result found in [33] at the same limit.

4. Conclusions

In this work we have used the approach of rationalizing the fermion propagator up to the leading order in the CPT-violating coupling b_μ, after using the derivative expansion method. We have shown that although the coefficient of Chern-Simons term can be found unambiguously in different regularization schemes at zero and finite temperature, it remains undetermined. An interesting point, however, we noted here is the possibility of the temperature making the connection among the coefficients obtained in different regularization schemes. In a certain sense, each value found in a particular scheme seems to correspond to a different scale of the theory, which can be achieved by increasing or decreasing the ambient temperature. For example, a particular coefficient found in zero temperature by adopting one approach corresponds to the same coefficient at high temperature by adopting another one. A complete understanding of these issues will require further investigations.

Acknowledgments

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